

Chapter 6

Relationships Within Triangles

6.1 Perpendicular and Angle Bisectors

6.2 Bisectors of Triangles

6.3 Medians and Altitudes of Triangles

6.4 The Triangle Midsegment Theorem

6.5 Indirect Proof and Inequalities in One Triangle

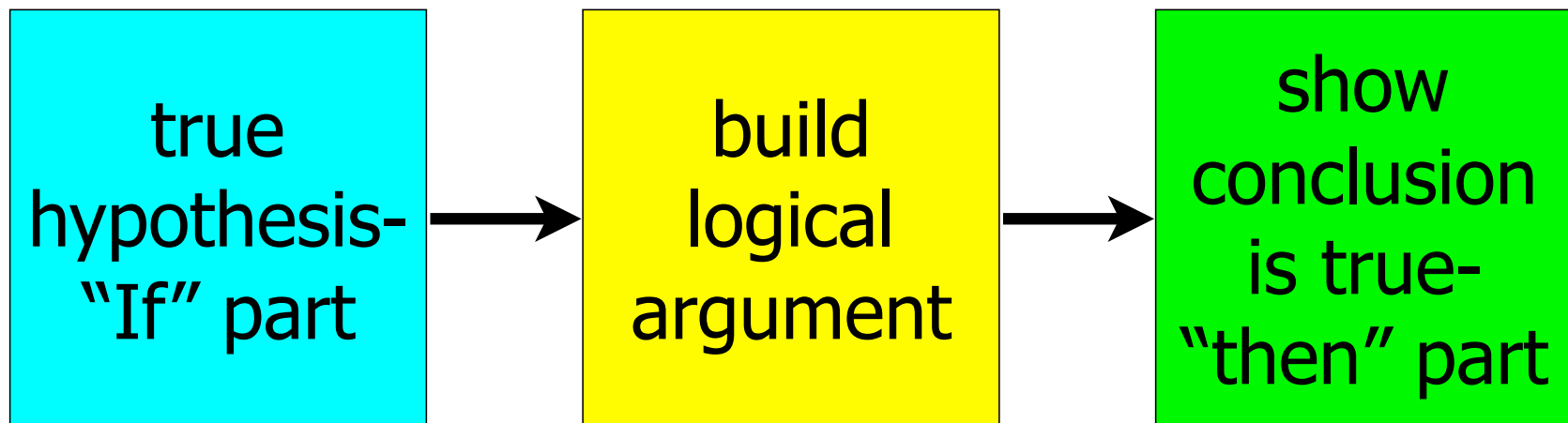
6.6 Inequalities in Two Triangles



6.5 Indirect Proof and Inequalities in One Triangle

So far we have written only **DIRECT Proofs**.

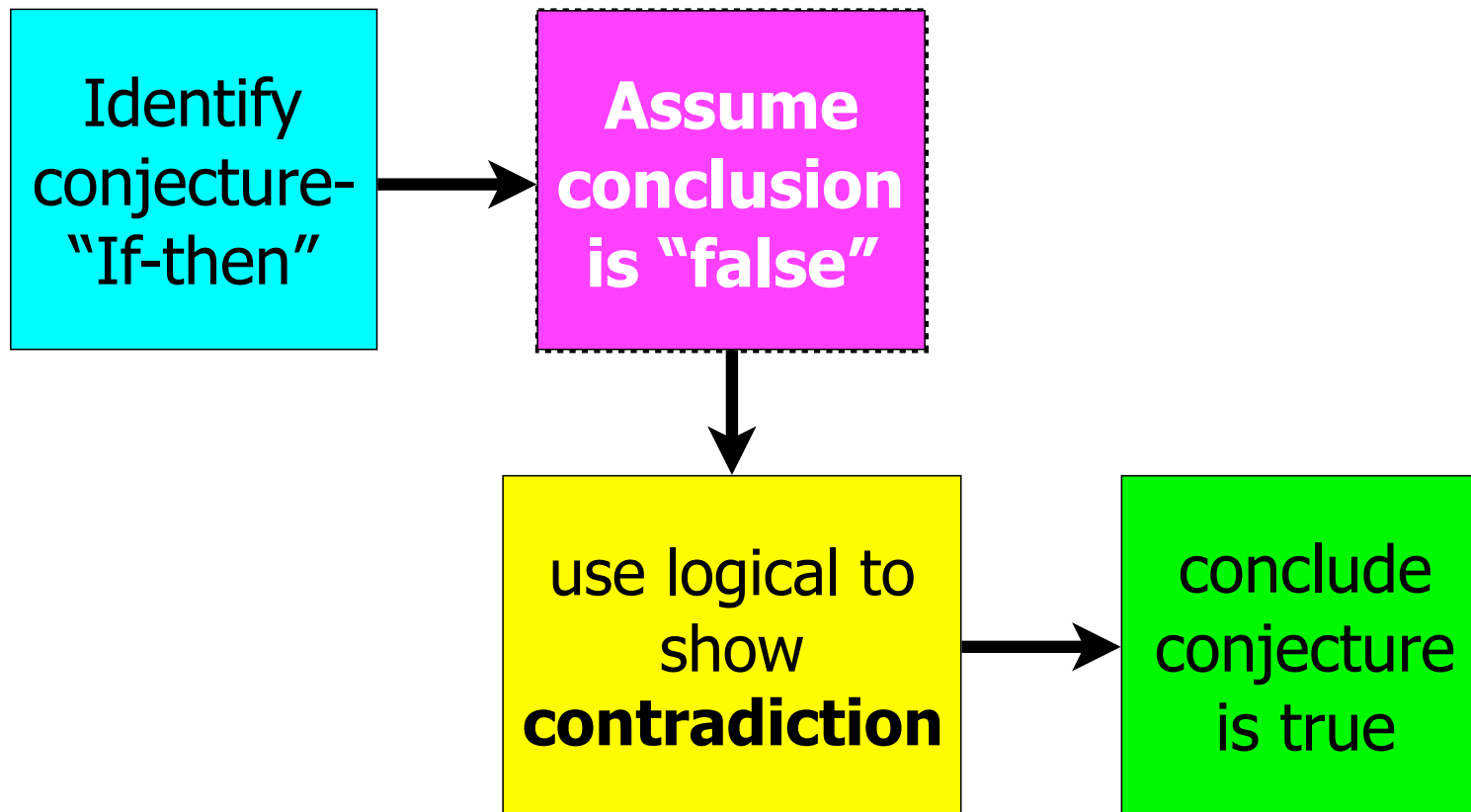
We did these with logical arguments.



6.5 Indirect Proof and Inequalities in One Triangle

INDIRECT Proof or “Proof by Contradiction”

Add one more step.



6.5 Indirect Proof and Inequalities in One Triangle

Given: Any triangle

Prove: A triangle cannot have two obtuse angles.

1.) Identify the conjecture to be proven:

If it is a triangle, then it cannot have two obtuse angles.

2.) Assume the opposite (negation) of the conclusion is true.

Assume, temporarily, a triangle CAN have two obtuse angles.

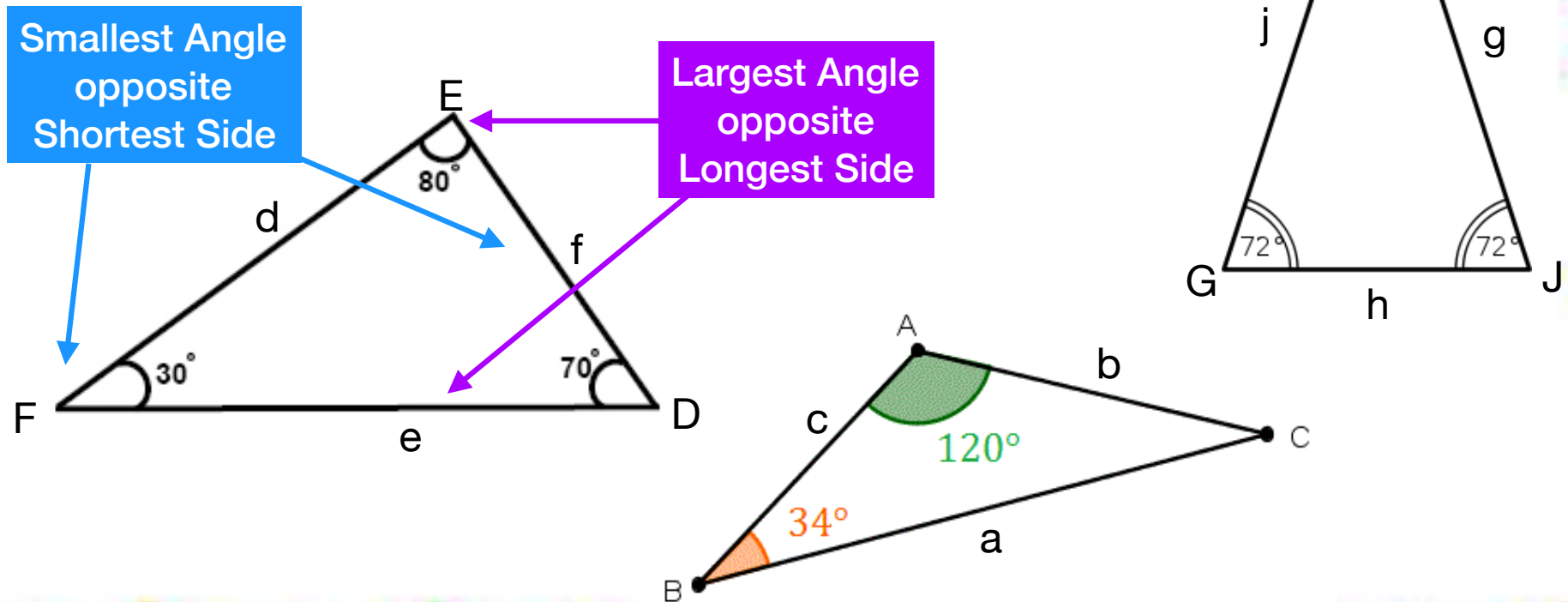
3.) Use direct reasoning to show that the assumption leads to a contradiction.

Assume, temporarily, that a triangle HAS two obtuse angles. The measure of any obtuse angle is greater than 90. So the sum of any two obtuse angles is greater than 180; which is a CONTRADICTION to the Triangle Sum Theorem. So my assumption is false.

4.) Conclude that since the assumption is false, the original conjecture must be true. ...**So my assumption is false, a triangle cannot have two obtuse angles.**

6.5 Indirect Proof and Inequalities in One Triangle

- Can you order the angles from smallest to largest?
- Using the angles, can you order the sides from smallest to largest?



6.5 Indirect Proof and Inequalities in One Triangle

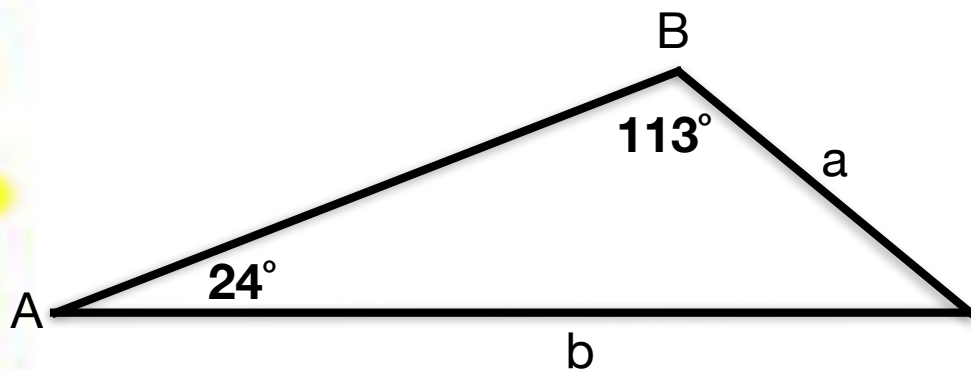
Theorems

Triangle Larger Angle Theorem

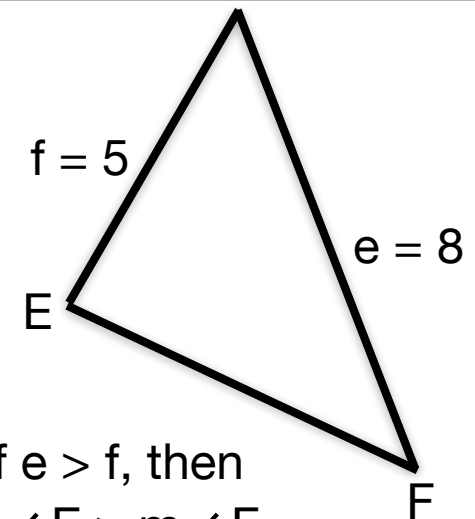
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Triangle Longer Side Theorem

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



If $m\angle B > m\angle A$, then $b > a$.

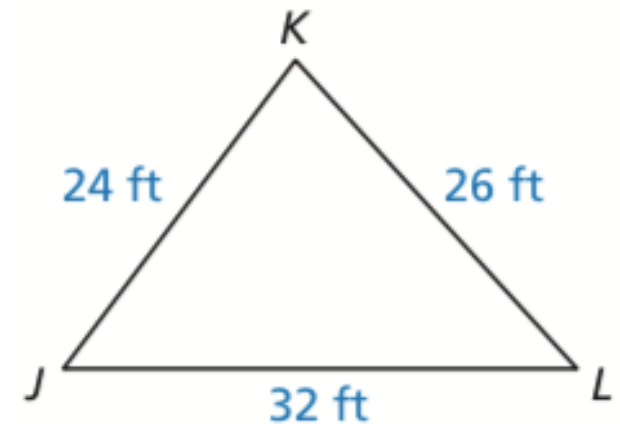


If $e > f$, then
 $m\angle E > m\angle F$.

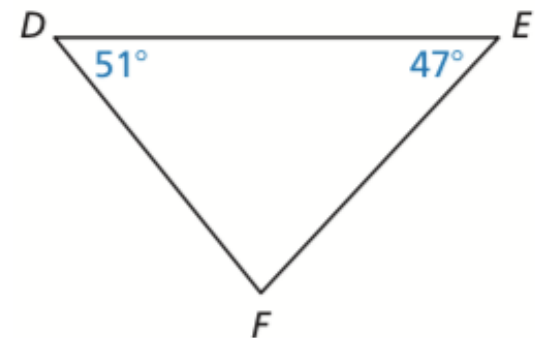
6.5 Indirect Proof and Inequalities in One Triangle

Exercise:

1) List the angles from smallest to largest.

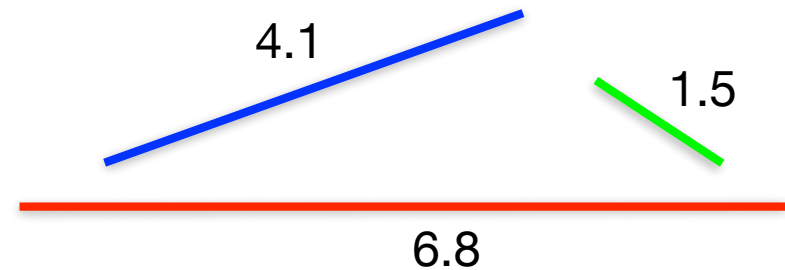
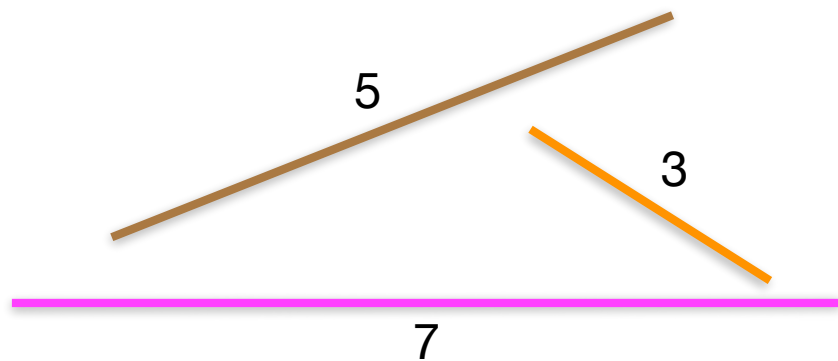


2) List the sides from shortest to longest.



6.5 Indirect Proof and Inequalities in One Triangle

- A triangle can be formed by 3 segments, but not every set of three segments will work.
- How are you supposed to know?



6.5 Indirect Proof and Inequalities in One Triangle

Theorems

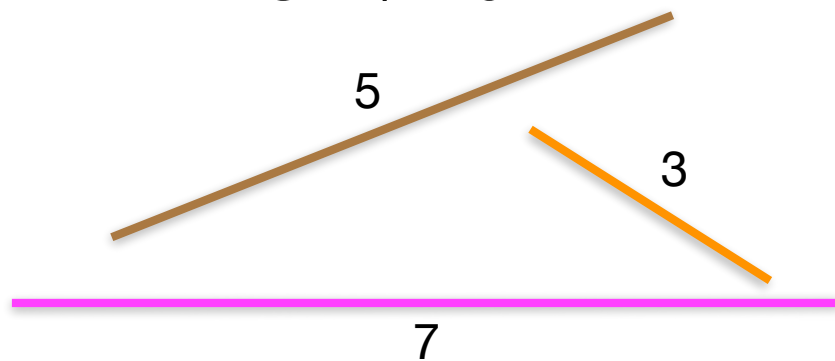
Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$5 + 3 > 7$$

$$5 + 7 > 3$$

$$3 + 7 > 5$$

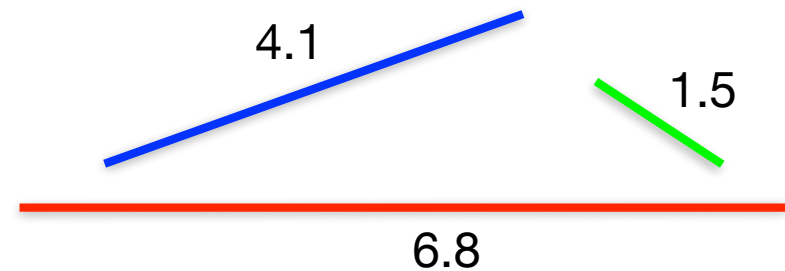


$$6.8 + 4.1 > 1.5$$

$$6.8 + 1.5 > 4.1$$

$$1.5 + 4.1 < 6.8$$

NOT a triangle



6.5 Indirect Proof and Inequalities in One Triangle

Exercise: Can you make a triangle out of these lengths?

a) 8, 12, 21

b) 6.2, 7, 9

c) 4.3, 5.7, 10

6.5 Indirect Proof and Inequalities in One Triangle

Exercise: The figure shows approximate distances. What is the range of distances from San Francisco to Oakland?

